Chapter 8

Search Theory

In this chapter, you will be introduced to a research area known as 'search theory'. In particular, we will focus on the Diamond-Mortensen-Pissarides model of labour search.[1] Before delving into that model, we will first look at what makes an issue a search problem and see how the analysis of search necessitates the use of methods from the study of stock-flow dynamics. We will also have a look at an extra problem associated with search theory, *viz* that of not having full/perfect information. The lesson here will be that once we investigate the problems accounted for in search theory, many results from standard, competitive equilibrium economics may not hold. Lastly, we will examine the Beveridge curve, which shows the relationship between vacancies posted by firms and the unemployment rate. The graphical analysis of the Beveridge curve will be helpful when it comes to understanding the Diamond-Mortensen-Pissarides model, taking the model to the data to assess its empirical validity and contemplating policy proposals, including those dealing with the recent global financial crisis.

[1] However, search theory can also be applied to studying housing markets and even the marriage market.

8.1 A Basic Search Problem

To see what characteristics define a search problem, let us imagine that Mr. Joe Bloggs wants to purchase a house along the seafront in Dublin.[2] Pretend that he is unhappy about the location he is currently living in and would like to move out sometime in the next year or so. Rather than making an immediate purchase – though this can happen sometimes – Joe would typically have to devote considerable time and resources to learn what possible choices he has. Since there are *many alternative opportunities*, at some stage, Joe would need to stop looking and make up his mind. Perhaps the first few hours of searching a website such as daft.ie would reveal a lot of information to Joe in helping him make an informed choice; however, after 2 years of searching, he would probably be very unhappy that he has not moved yet. At each point in time, he might find a better place to buy, but he will also try to form expectations about what benefits another day of searching might bring and about the cost of spending another day living in his current home in which he is unhappy. Mathematically, we could write down these expectations of benefits and costs as part of a sequence over time – this is the *sequential* aspect of a search problem: he will have to *stop* searching at some stage. Already, lots of time is necessary to analyse such a purchase. Finally, with all this time and energy, Joe is probably not planning to move out very soon or buy very soon, but rather Joe is looking at future benefits and costs, which are uncertain, so *expectations* come into play. So, we have seen that a search problem involves alternative opportunities that must be assessed. We have seen that a search problem involves breaking a problem into sequences of time where we have to decide when to stop searching. And we have seen that uncertainty and hence expectations play a role.

Stock-Flow Dynamics

Let us turn next to looking at the relation between *stock-flow dynamics* and search theory. A stock is a quantity defined at a point in time, e.g. the amount

[2] As another example, consider the situation of choosing a dress or suit to wear to a wedding of a distant acquaintance who has invited you as a guest.

of money you have in a savings account. A flow is a quantity that is defined over an interval of time, e.g. the income you earn each month. You could save this income and thereby raise the amount in your savings account or take out some of your savings and so reduce the amount in your savings account. So, a flow variable can be used to *change* the value of a stock variable.

Stock-flow dynamics are central to labour search theory. For instance, while unemployment and vacancies coexist, plenty of people find jobs and plenty of others lose them: flows into and out of the labour market are sizeable. Unemployment can exist as an equilibrium outcome in such models. The level of unemployment at a given point in time is a stock that tends towards a level that balances the inflows to and outflows from the labour market.

People tend to spend a short time looking for work – though longer during recessions – and longer times employed once they find jobs.[3] The labour market is always flowing, so flows matter. Unsurprisingly, flows tend to be smaller during recessions as the correlation between hires and separations is high and procyclical.

Information Frictions

Economists love to use the term frictions to describe departures from when everything is idyllic and things run smoothly. This terminology derives from the way that oil is used to prevent overheating caused by the friction of moving parts in an engine. For example, when prices are not perfectly flexible, we could describe the situation as one in which there are price frictions. The point is that there is something that prevents things from running smoothly.

With price setting in an environment with search frictions (things preventing search from being effortless and immediate), it turns out that prices move very far from the equilibrium without search frictions. For instance, consider the case of going to a store to buy a t-shirt where you see a price, but there is a cost to going to another store (e.g. time and cost to travel to next store). So the store you are in has a little bit of market power: it knows that it can charge a slightly higher price for the t-shirt and that you may not know that

[3] Having said this, lots of people change their jobs, rather than become unemployed.

you could go to a shop next door where the price for the same t-shirt is slightly cheaper. It turns out that the price in equilibrium is the price a monopolist would charge if a monopoly owned all the outlets of the store. The reason behind this result is that if all outlets charged the same price, then each could charge a little more but not too much (not more than the cost of you going to another store) and therefore raise the price and profit. This rise in prices only stops at the monopoly price. So, instead of the law of one price where the same price is offered, there can now be more than one price offered, so there can be a distribution of prices within a market. Furthermore, prices are greater than marginal cost. So, search could make a large change to the standard equilibrium. A tiny search cost will make a big difference, whereas when the search cost is zero, you get the competitive equilibrium, so there is a discontinuity (a big difference in outcomes for small difference in the degree of frictions), which is a paradox within the search literature.[4]

Moving on to labour, rather than looking at wages, workers can look for types of jobs and firms can look for worker types. You may want a specific job such as a business studies teacher in a mixed, non-denominational secondary school in a disadvantaged area. Likewise the school may be looking for someone with effective communication skills, good control of students and a proven record in business with knowledge as demonstrated by course work and / or real life experience. The bargaining tool between the worker and the school will be the wage. The worker will be willing to take a slightly lower wage if s/he gets the type of job s/he wants and likewise, the school will be able to offer a slightly higher wage if the candidate possesses the skills that the school is seeking in an employee. So, the prices or wages are shared and the outcome results from a bargaining game, *viz.* the Nash wage condition.[5] Frictions also

[4] *Heterogeneity* is a complication that has received a lot of renewed interest over the past few years. In particular, this refers to different types of buyers and sellers, different ways of learning about prices, etc.

[5] Named after the Nobel Laureate John Nash, the same person as that portrayed by Russel Crowe in the film 'A Beautiful Mind', the Nash wage condition is a solution to a bargaining problem that satisfies certain technical conditions. In particular, the solution should not depend on different utility functions (where they can be shown to be equivalent); the solution should be Pareto Optimal (deviating from the solution should make at least one person worse off); eliminating some of the unchosen alternative solutions should not affect

exist such as the mismatch between skills employers are looking for and skills possessed by workers, differences in location, institutional structure, transmission of information about jobs, etc. The matching function, discussed below, summarises these frictions.

Beveridge curve

The *Beveridge curve* describes the relationship between vacancies and unemployment. High unemployment is typically associated with low vacancies and *vice-versa*. Moreover, outward shifts of the Beveridge curve imply decreasing labour market efficiency. The labour market becomes relatively less efficient at matching workers with jobs when there are more vacancies for any given level of unemployment. While the Beveridge curve was named after Lord William Beveridge, his original analysis in a 1944 report did not include any graphical representation of the relationship between unemployment and vacancies.[6]

Unemployment insurance that makes workers less accepting of jobs tends to push up the Beveridge curve, while some unemployment insurance that supports the unemployed during recessions but a the same time provides incentives for more intensive job search can increase the efficiency of job search. This latter type of unemployment insurance is part of what is known as 'active' labour market policies; its opposite is sometimes referred to as 'passive' labour market policies.

UK unemployment rose in the 1980s though vacancies remained approximately constant, indicating more frictions in the labour market, or a horizontal movement outwards in the Beveridge curve. Frictions became stronger since the British economy was transforming from a manufacturing economy to a service economy, so there was more mismatch between workers and jobs. A

the selection of the solution as the best option ('independence of irrelevant alternatives'); the symmetry axiom holds. On this last axiom, a bargaining problem is symmetric if the following two conditions hold: (i) the utility from disagreeing is the same for the worker and the firm and (ii) if the worker gets a utility of u_1 and the firm gets u_2, it must be possible to also get u_2 and u_1 for the worker and firm, respectively. If the bargaining problem is symmetric, then the bargaining solution will be the same for worker and firm (they both disagree and get the same utility or they both agree on the same wage). See Nash (1950) for more.

[6] Unfortunately, data is not a great proxy for vacancies and unemployment.

rise in long term unemployment brought with it the disillusionment of unemployed workers and reduced their incentive to work. This hysteresis effect is an extra cost of recessions, detrimental to the labour force and tends to prolong recessions. Unemployment did not fall even though there was an economic boom in the late 1980s. The reduction in long term unemployment eventually was achieved through active policies.

Job destruction and creation are cyclical and depend on the reservation wages and profitability/productivity. Employment legislation has the aim of making job destruction more costly and lengthening the time in a job. Such legislation differs across countries – though it is stricter in southern Europe – with taxes on dismissal reducing job separation, but in turn failing to destroy low productivity jobs. Average productivity falls and wages fall too to compensate firms for taxes and lower productivity. Job creation is reduced and the net impact on unemployment depends on which flow falls more: flows into or out of unemployment, unemployment falls to compensate and vice-versa. Labour, turnover and productivity fall and the duration of both unemployment and employment rises. Employment protection may help encourage training so workers may be employed longer, they are more secure in their jobs and become more willing to undertake training that is specific to the needs of the firm. Employment protection legislation tends to affect different types of workers differently. For instance, such legislation typically benefits primary workers (men over 25) but hurts women and youths since these types of workers have more flows into and out of unemployment.

8.2 Diamond-Mortensen-Pissarides model

Assume there are L people in the labour force – the labour force consists of those persons employed or unemployed but available for and seeking work. From here on, when I say unemployed, I implicitly will mean those who are available for and seeking work. The *unemployment rate* is $u = \frac{U}{L}$ and the *vacancy rate* is $v = \frac{V}{L}$, where $U = uL$ is the number of unemployed workers in the economy and $V = vL$ is the amount of vacancies for unfilled jobs posted by employers. The total number of unemployed workers is uL and the total

number of employed workers is $(1-u)L$. Let M be the total number of matches between firms and workers. Then assume the following functional form for the relationship between the number of matches and the number of unemployed workers and vacancies in an economy:

$$M = xU^{\alpha}V^{1-\alpha} = x(uL)^{\alpha}(vL)^{1-\alpha} = xLu^{\alpha}v^{1-\alpha}$$

where x is a stochastic matching shock, akin to those in the RBC model. For example, x could rise with say the advent of the internet. Since the internet improve the scope for advertising vacancies and increased efficiency in matching unemployed workers with vacant, unfilled jobs, M would rise through an increase in x, which represents the development of the internet. The *job-finding rate* is

$$\frac{M}{U} = \frac{xLu^{\alpha}v^{1-\alpha}}{uL} = x\left(\frac{v}{u}\right)^{1-\alpha} = xa(\theta)$$

The ratio $\theta = \frac{v}{u}$ is a measure of *labour market tightness* and is central to the DMP model. The *vacancy-filling rate* is

$$\frac{M}{V} = \frac{xLu^{\alpha}v^{1-\alpha}}{vL} = x\left(\frac{u}{v}\right)^{\alpha} = xq(\theta)$$

Jobs are destroyed at rate ϕ, so the change in employment is

$$\dot{U} = \underbrace{\phi(1-u)L}_{\text{Flows Into Unemployment}} - \underbrace{xauL}_{\text{Flows Out of Unemployment}}$$

In equilibrium, there is no change in employment, i.e. $\dot{U} = 0$:

$$\phi(1-u)L = xauL \Longrightarrow u = \frac{\phi}{\phi + xa(\theta)}$$

With labour market equilibrium, free entry implies that jobs are created until the marginal vacancy has zero value, which implies that there are less vacancies per unemployed worker when the wages promised workers are high. This is the main story behind the demand curve. With the supply curve,

the bargaining outcome equation is such that workers and employers share the surplus value of a match so that wage demands increase when the $\frac{v}{u}$ ratio increases since it becomes easier to find jobs. A *search equilibrium* is a combination of a $\frac{v}{u}$ and a wage w such that the forces of supply flows and demand flows are balanced. Mathematically, a search equilibrium is defined as

$$\left(\frac{v}{u}, w\right) : \dot{U} = 0$$

Regarding what determines the positions of the curves, the location of the wage equation depends on unempoyment insurance, for instance. Furthermore, the job creation curve depends on expectations of workers and employers for the future. These expectations concern whether the product can be sold on markets, so for instance if expectations are optimistic, the demand curve shifts out, but if expectations are negative, then the demand curve shifts inwards.

During the Great Recession, housing prices fell, which affected labour supply as house prices are a source of wealth. Houses were then used to support consumption, e.g. borrowing to buy automobiles, etc. As demand contracted, expectations were that future demand would be lower too. So, this shifted the job creation curve inwards since the future looked more pessimistic. Then there were further shifts down in the job creation curve so that the intersection point fell. The pressure on wages fell and there was a major drop in vacancies. In tandem, it would take longer to find jobs so unemployment increased.

This prediction is empirically verified by looking at the US Beveridge curve from about 2002-2010. During recessions, there are movements down the Beveridge curve and then up as job creation first is hit by negative expectations regarding the future. In 2007, unemployment rose. From June 2008 to June 2009 unemployment plummeted futher and vacancies plunged. The model explains what went wrong, *viz.* expectations of the future tanked. These expectations dropped because workers and firms acted as if there were a reduction in the value of matches and this continued.

Regarding policy implications, expected future flows need to be discounted through interest rates. One problem faced by employers is financing their

payroll – particularly small business – not because the FED has set high rates (they are low), but actually because bankers are scared to lend to small firms and so effectively the rates are high. It is not the case that loans are expensive in terms of interest rates, but rather firms find it difficult to obtain loans in the first place. Fiscal tightening in the US and Europe will not help before employment returns to normal, though it something we need to do in the long run. If we can get employment to return to normal, then we can increase government revenues so that fiscal budgets can be balanced. Extending unemployment benefits may help, but we will need to do more. We have available resources, i.e. lots of unused workers, so it is a good time to invest. However, businesses are pessimistic, so they will not necessarily be the solution to the recession. We can invest in education, infrastructure and the environment and making use of these resources will be good for the economy and fiscally. This investment may require more borrowing. However, it is still the case that the US needs to attack long run issues over fiscal balances, for instance not financing another war with a tax debt.

The theory of wages is one way to understand the labour markets. Modern theory needs to develop more along the line of the role of institutions. Wage stickiness is very important in macroeconomics today, with more and more models containing different types of frictions. Rational expectations and perfect captial markets were a benchmark, but implied that we could borrow an infinite amount of money. Integrating the financial sector with labour market frictions may explain why shocks get amplifed as we see in the data.

Recent Debate on the Empirical Performance of DMP Model

The DMP framework, with the addition of aggregate productivity shocks, has been the dominant framework for macro analyses of the labour market in recent years. Its advantage over frictionless models is that the DMP model accounts for the simultaneity of unemployment and vacancies and of job creation and job destruction. However, a few years ago Robert Shimer questioned the ability of the DMP model to match the cyclical behaviour of unemployment, vacancies, job creation and job destruction, calling the usefulness of the model

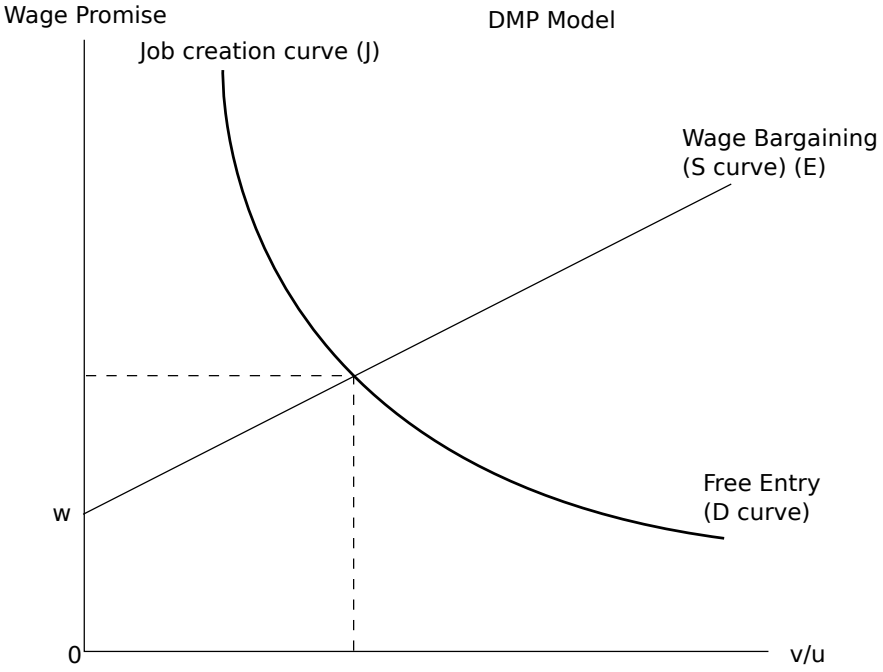
into question. Since then, there has been a big debate on changes to the model to match the data, or alternative frameworks for labour market analysis.

Shimer (2005) uses a simple version of the DMP model with aggregate shocks to producitivity. He chooses the model parameters to match (a) the standard deviation and serial correlation of labour productivity (b) the average replacement rate of 40% (i.e. b, the current period monetary value of the utility of being unemployed is 40 percent of the average wage) (c) the average interest rate (d) quarterly separation rate (e) average job-finding rate (f) average vacancy to unemployment ratio (g) estimated elasticity of matching function (from Beveridge curve). The central research question he asks is whether the estimated model can match the volatility of unemployment, vacancies, the job-finding rate and the separation rate. It turns out that like the equity premium puzzle and CAPM, volatilities here are much smaller than in the data. Shimer goes through some variations of the model, but the results stay basically the same. Shimer's conclusion is that the problem is not the matching model, but the assumption of Nash bargaining. Changes in wages absorb most of the fluctuation in productivity, so that v, u, etc. do not need to do much. He proposes that other wage-setting mechanisms should be explored.

Hall (2005) argues that there is really no good reason to use Nash bargaining since any wage-setting solution that gives at least some of the surplus to each party is a legitimate solution. Let's replace wage bargaining with a sticky wage assumption: (i) wage stays constant as long as the surplus for the worker and the firm remain positive; (ii) adjust the wage by the minimum required amount when the surplus for one party turns negative. Stickiness could perhaps be justified with bargaining costs. Hall focuses on the case with small shocks where the wage never has to be changed. The intuition for the larger effects is the following. When productivity falls, given a constant wage, the share of surplus for the firm falls rapidly, while the worker's share goes up. The declining surplus for the firm means that the probability of finding a worker for a firm has to increase in order to compensate for the lower surplus and satisfy the free entry condition. This can be achieved only by a large change in $\frac{v}{u}$.

A number of authors have argued that Shimer and Hall's conclusions regarding the wage-setting mechanism may be premature. Hagedorn and Manovskii (2008) focus on the calibration of the model. Shimer and Hall set $b = 0.4$, based on unemployment compensation, e.g. unemployment benefits. But since b should represent the full utility of unemployment (including leisure, return to home production, etc.), its true value may be higher. They check whether b can be chosen such that the model matches data even with Nash bargaining. The calibration matches the variability of wages to the data. So, the model matches the data fairly well. However, others have criticized the high value for b. It implies, among other things, that small changes in unemployment compensation should have large effects on equilibrium unemployment.

A number of authors argue that what really matters is what happens to new jobs. For example, Eyigungor (2009) builds a model with vintage-specific productivity and specific capital. This model makes productivity of new matches more volatile, leading to a volatile labour market even with standard parameters. Braun (2005) incorporates turnover costs (i.e. hiring and firing costs or training costs) that have a similar effect as a higher b since the surplus is diminished and there is a smaller interval for the wage to move in. This allows matching the data with more realistic values for b. Finally, Nagypal (2005) allows for job-to-job transitions. These transitions are highly cyclical and make up for more than half of the hires, which helps too.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DMP Model
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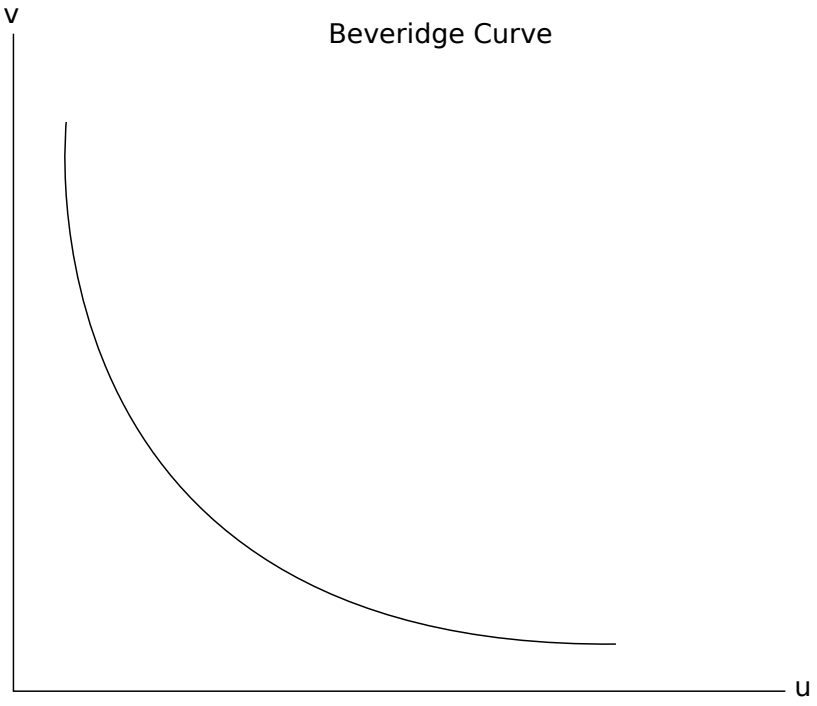
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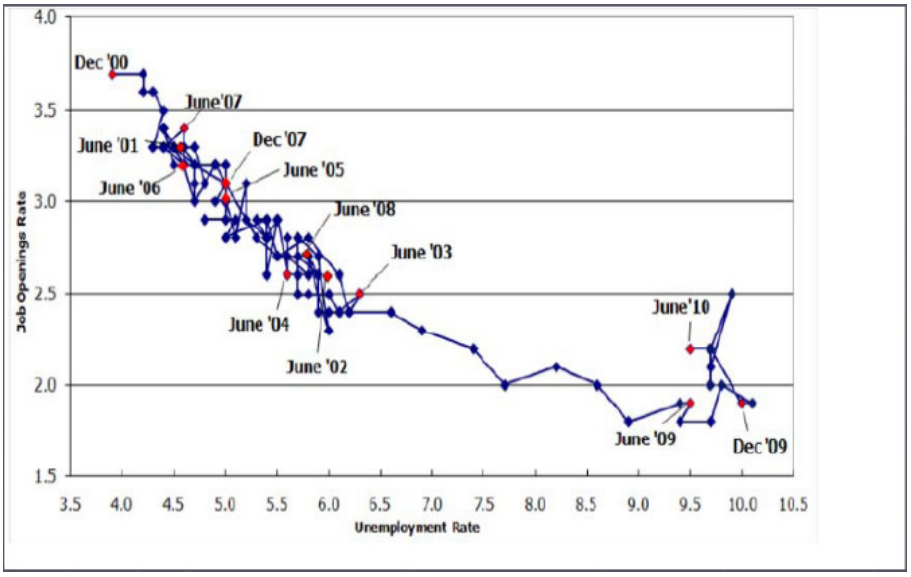
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Job Openings Rate
Unemployment Rate
4.0
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